Spatial locations

Where am I?

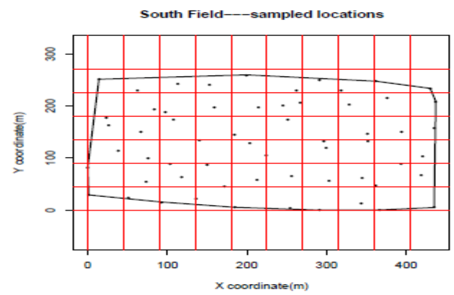
- sounds simple, but definitely not
- huge amount of picky detail
- Cartographers make careers from the details
- lecture focuses on the minimum you need to know

Spatial locations

Sometimes a non-issue

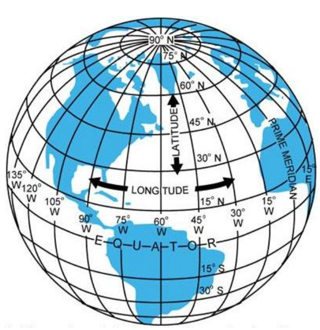
- Measuring soils in a grassland. Set out a grid using meter tapes
- Arbitrary (0,0) point
- Distance is obvious: Euclidean

$$D_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$



Spatial locations

Latitude - Longitude



Spatial locations

Latitude - Longitude

- scheme developed and adopted in 1890's
- Latitude: 0° is the equator, increase towards either pole
- Longitude: 0° passes through Greenwich, England.
  - W towards US, E towards Asia
  - 180° W = 180° E in middle of Pacific Ocean (mostly)
- units are degrees, minutes, seconds
- my house in Ames: $42^\circ\ 3'\ 20''$ N, $93^\circ\ 38'\ 10''$ W
- Decimal degrees: my patio: long: -93.63611°, lat: 42.05556°
- -: W or S, + E or N

Spatial locations: Greenwich Prime Meridian

Spatial locations

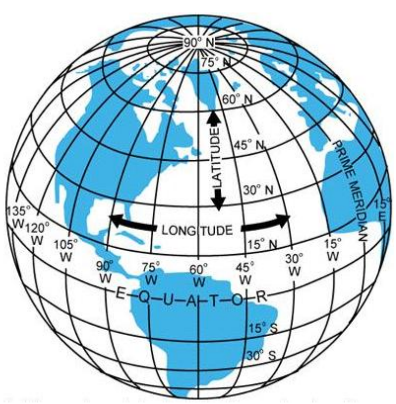
Distance in a Latitude - Longitude coordinate system

- How far from my house to my office?
  - my house: long: -93.63611°, lat: 42.05556°
  - my office: long: -93.6333°, lat: 42.01667°
  - Calculate Euclidean distance using lat long values
  - A change of 0.00278°, 0.0389°
  - So they are: $\sqrt{0.00278^2 + 0.0389^2} = 0.0390$ degrees apart
- Convert to km (or miles):
  - 1° latitude = 60 nautical miles, 69.047 statute miles, 111.12 km
- 2.69 miles, 4.33 km.
- Wrong! almost always

Spatial locations

Distance in a Latitude - Longitude coordinate system

- Problem is that 1° lat. is longer than 1° long.



Spatial locations

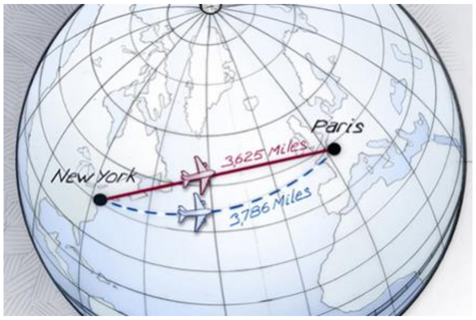
Distance in a Latitude - Longitude coordinate system

- Examples:
  - At 42°N: 1° long. ≈ 110 km, 1° lat. ≈ 84km
  - On equator: 1° long. ≈ 110 km, 1° lat. ≈ 110km
  - At 35°S: 1° long. ≈ 110 km, 1° lat. ≈ 90km
  - At 50 S: 1° long. ≈ 110 km, 1° lat. ≈ 70km
  - In general: 1° long. at lat. $l = 111.12 \cos l$

Spatial locations

Distance in a Latitude - Longitude coordinate system

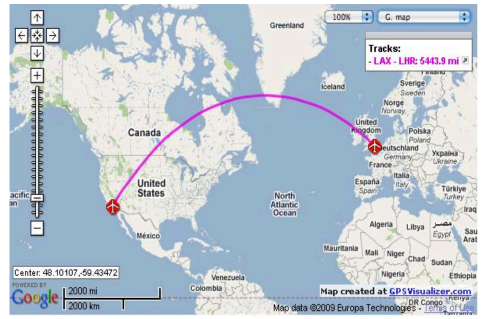
- So how to measure distance on the sphere?
- Great Circle distance:



Spatial locations

Great Circle distance

- Straight lines on the globe become curves in a plane



Spatial locations

Great Circle distance

- Given two points on a sphere,
  e.g., (ϕ_1, λ_1) and (ϕ_2, λ_2) in decimal lat long
- Use spherical trigonometry:

$$d = r \cos^{-1} \left(\sin \phi_1 \sin \phi_2 + \cos \phi_1 \cos \phi_2 \cos(\lambda_1 - \lambda_2) \right)$$

- Use a computer to do the calculation.
- Other formulae avoid the difference of two potentially small numbers.
- Take home: distance can be calculated without converting locations to points on a plane.

Latitude Longitude:datum

Locations on the sphere

- One more issue:
  - GPS says my house is at long: -93.63611°, lat: 42.05556°
  - an old map says: -93.63589°, lat: 42.05557°
  - that is 18.1m apart. Other places 100+m apart
- The issue is the "datum": math model of the world

Latitude Longitude: datum

Locations on the sphere—

- the Earth is not a perfect sphere
- locations computed using a mathematical model of the world
- that model has changed over time
- GPS units use WGS84 datum
- Newer US maps use NAD83 datum, now reconciled with WGS84
- Older US maps use NAD27 datum
- Even older maps: be very careful
  - late 18th century map of Atlantic Ocean:
  - 0 longitude went through Dakar, Senegal
  - (Now put at 17.44 ° W)

Consequences of datum choice

Results in two different datum's have a frameshift

- Changes between locations are precise within each datum
- Absolute locations change between datum's
- The difference can really matter
  - European example: finding a harbor entrance in the fog
  - The "Royal Netherlands Navy" pamphlet.
  - Details were in 1st ed. of Bivand. Bits are in the 2nd ed. p. 86, 88
  - map (marine chart) used ED50 datum for lat. long.
  - ship navigation used GPS system to get lat. long.
  - Location on marine chart (ED50) is 124m off of where a GPS (WGS84) says it is
  - Ship ran aground.

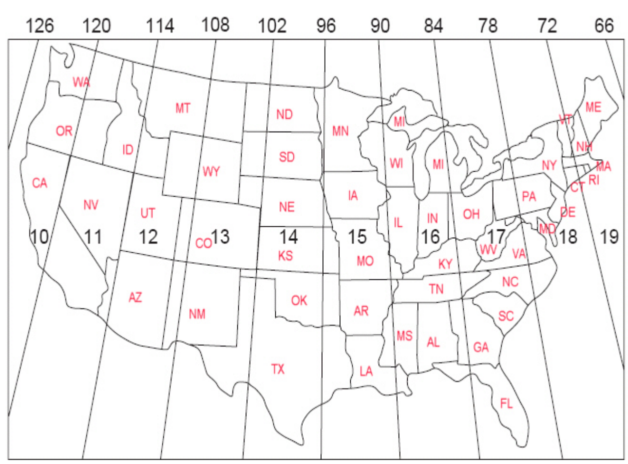
Converting lat long to (X, Y) coordinates

How to set out a (X,Y) coordinate system for large distances?

- problem: a sphere is not a plane
- all maps are distortions
- a good atlas does a lot of work to minimize these
- (Which is the heavy technical stuff in geodesy and cartography)

Usual solution is a UTM coordinate system:
Universal Transverse Mercator

- Divide world into 120 zones (60 N, 60 S), each 6° long. wide
- UTM zone 15N from -90° to -96°, includes almost all of Iowa
- Sliver in extreme west in zone 14N (Sioux City area)



UTM coordinates

- units are meters EW and meters NS
- middle of zone, e.g. -93°, arbitrarily set at 500,000
- deviations E or W are "Eastings"
  - UTM coordinates < 500,000 are W of midline
  - UTM coordinates > 500,000 are E of midline
- "Northing" is m N of equator
- Presume "Southing" is m S of equator
- my house is 447,346.4 E, 4,656,139 N
- Easy to calculate distances from UTM's
  - Euclidean distance formula
  - so long as both points are in the same zone

Where is Omaha?

- Omaha NE: 41.3°N, 95.9°W
  - On the E edge of zone 15 (boundary is 96°W
- If calculate UTM in zone 15: (257,201.2E, 4,576,118N)
- If calculate UTM in zone 14: (759,545.6E, 4,576,697N)
  - Big difference is from E edge of zone 15 to beyond W edge of zone 14
  - Can adjust by subtracting width of zone
- Omaha at either (257,201E, 4,576,670N) (zone 15)
  or (257,112E, 4,576,118N) (zone 14)
- Just over 0.5km apart!
- Again: sphere ⇒ plane issue
- Better to stick with one zone and accept a bit of distortion at the edges.
- Or, just use Great Circle distance

Does it matter whether use Euclidean or GC distance?

- Reminder:
  - Rectangular coordinate system (e.g. UTM): Euclidean distance
  - length of shortest path between two points on a plane
  - Spherical coordinate system (e.g. Latitude/longitude): great circle distance
  - length of shortest path between two points on a sphere
- How much does it matter?
- Next page: distance between Ames IA and different IA county seats
- Difference between GC on long/lat and ED on UTM coordinates is small (< 20m for 50 km distance)
- a bit more ($\approx 60m$ for 250 km distance
- My advice: difference not important for < 250 km, might be for large-scale projects
- If so, use great circle distances, especially if crossing UTM zones.